

Rep Thy I: Problem Set 6 (due Mon Nov 19)

Notation: In Problems A and B, $G = \mathrm{GL}(2, q)$ with $q = p^r$ (p prime, $r \geq 1$), B be the (standard) Borel subgroup, U the (standard) unipotent subgroup and D the diagonal subgroup.

- Exercises 1–5 from notes on Real, Complex and Quaternionic Representations
- **Problem A:** Let $X = \mathrm{Hom}(U, \mathbb{C}^\times)$ be the group of irreducible characters of U .
 - (i) For $\delta \in D$, $\psi \in X$, let $(\delta \cdot \psi)(u) = \psi(\delta^{-1}u\delta)$. Determine the D -orbits of X with respect to this action.
 - (ii) Determine the number of irreducible representations of B of each dimension, using Proposition 25 of Serre.
- **Problem B:** Prove the Bruhat decomposition $G = B \sqcup BwB$, where $w = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$.

Presentations

WL (Ex 1), JD (Ex 2), EH (Ex 3), JL (Prob A), JR (Prob B)