Calculus III Honors Spring 2012
Exam 1 - Practice Problems, Set 2

1. T F If (a_n) is increasing and $a_n < 1$ for all n, then $\lim_{n\to\infty} a_n = 1$.
2. T F If $\sum a_n = \infty$ and $\sum b_n = \infty$, then $\sum(a_n + b_n) = \infty$.
3. T F If $a_n \to 0$ as $n \to \infty$, then $\sum a_n$ converges.
4. Define what it means for a series to converge. No!

For each of the following problems, determine (a) if the sequence (a_n) converges, and if so, find its limit; and (b) if the series $\sum a_n$ converges, and if possible, find its limit.

5. $a_n = \cos(n\pi/2)$ (a) diverges (b) diverges
6. $a_1 = 5$, $a_{n+1} = a_n/3$. (a) $\to 0$ (b) $\to \frac{5}{1-1/3} = \frac{15}{2}$
7. $a_n = \frac{n^n}{n!}$ (a) $\to \infty$ [diverges] (b) diverges
8. $a_n = \frac{2}{n^2-2n}$, $n > 2$ (a) $\to 0$ (b) $\sum_{n=3}^{\infty} a_n = 2\sum \frac{1}{n(n-2)} = \sum_{n=3}^{\infty}\left(\frac{1}{n-2} - \frac{1}{n}\right)$
   $= \left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \cdots$
   $= 3/2$
9. $a_n = \left(\frac{1}{n} - \frac{1}{n+1} + \frac{1}{n+2}\right)$.
10. $a_1 = 1$, $a_{n+1} = \sqrt{1 + a_n}$.

9\. (a) $\to 0$ (b) $\left(1 - \frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{4} + \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{5} + \frac{1}{6}\right) + \cdots$

$= 1 + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots = \sum \frac{1}{n} - \frac{1}{2}$ diverges

10\. (a) note a_n is increasing (at least note $a_n > 0$).
if it has a limit L, it must satisfy
$L = \sqrt{1+L} \Rightarrow L^2 = 1 + L \Rightarrow L^2 - L - 1 = 0$
$\Rightarrow L = \frac{1 \pm \sqrt{5}}{2}$ Since $L > 0$, have $L = \frac{1+\sqrt{5}}{2}$.

To check limit exists, can show by induction
a_n is increasing & $a_n < \frac{1+\sqrt{5}}{2}$ for all n.