

Here are some review problems I recommend you try **before class Fri Nov 16**, which you may solve by any method we've covered so far.

Disclaimer: While there are more questions on here than will be on the exam, not all methods/concepts/topics you might need/want to use on the exam are necessarily covered here. I chose the questions here somewhat at random, and many if not all of the problems can be solved in multiple ways.

1. Prove or disprove: Every odd integer is the sum of three odd integers.
2. Prove or disprove: if $x, y \in \mathbb{R}$ and $|x + y| = |x - y|$ then $y = 0$.
3. Prove or disprove: for $a, b, c \in \mathbb{Z}$, if $a|bc$ then $a|b$ or $a|c$.
4. Prove or disprove: there exist $a, b \in \mathbb{Z}$ such that $a^2 + 4b = 3$.
5. Prove or disprove: if A, B, C are sets, then $A \times (B - C) = (A \times B) - (A \times C)$.
6. Prove or disprove: if A, B, C are sets, then $A - (B \cup C) = (A - B) \cap (A - C)$.
7. Prove that if n is odd, then $8|(n^2 - 1)$.
8. Prove that $\gcd(n, n + 1) = 1$ for all $n \in \mathbb{N}$.
9. Prove that $n^3 \equiv n \pmod{3}$ for $n \in \mathbb{N}$.
10. Prove that for all $n \in \mathbb{N}$, $4 \nmid (n^2 + 2)$.
11. Prove $2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 2$ for $n \in \mathbb{N}$.
12. Prove $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for $n \in \mathbb{N}$.
13. Prove that $\{12n : n \in \mathbb{Z}\} \subseteq \{2n : n \in \mathbb{Z}\} \cap \{3n : n \in \mathbb{Z}\}$.
14. Prove that $\{4k + 5 : k \in \mathbb{Z}\} = \{4k + 1 : k \in \mathbb{Z}\}$.