

Here are some comments about how to do these problems/where to find them in the book—Ch  $n$ , Ex  $m$  means Exercise  $m$  of Chapter  $n$ . Since the book has solutions for odd numbered exercises, I'll mostly only make some comments about the even numbered ones.

1. Prove or disprove: Every odd integer is the sum of three odd integers.

See Ch 9, Ex 15.

2. Prove or disprove: if  $x, y \in \mathbb{R}$  and  $|x + y| = |x - y|$  then  $y = 0$ .

See Ch 9, Ex 29.

3. Prove or disprove: for  $a, b, c \in \mathbb{Z}$ , if  $a|bc$  then  $a|b$  or  $a|c$ .

See Ch 9, Ex 25.

4. Prove or disprove: there exist  $a, b \in \mathbb{Z}$  such that  $a^2 + 4b = 3$ .

This is a reformulation of Ch 6, Ex 7.

5. Prove or disprove: if  $A, B, C$  are sets, then  $A \times (B - C) = (A \times B) - (A \times C)$ .

This is a reformulation of Ch 8, Ex 18. You can use what we called Method 1 or Method 2 in class.

6. Prove or disprove: if  $A, B, C$  are sets, then  $A - (B \cup C) = (A - B) \cup (A - C)$ .

This is Ch 9, Ex 8. Suggestion: look at a few examples, e.g., what happens if  $B \cup C = A$ ?

7. Prove that if  $n$  is odd, then  $8|(n^2 - 1)$ .

This is Ch 5, Ex 17.

8. Prove that  $\gcd(n, n + 1) = 1$  for all  $n \in \mathbb{N}$ .

See Ch 7, Ex 33. (Suggested method: contradiction.)

9. Prove that  $n^3 \equiv n \pmod{3}$  for  $n \in \mathbb{N}$ .

See Ch 7, Ex 10. Suggestion: rewrite as  $3|(n^3 - n) = (n - 1)n(n + 1)$  and show 3 has to divide one of the factors on the right (e.g., you can use cases). One can also do it by induction, e.g., like Ch 10, Ex 13 which is the same with 3 replaced by 6.

10. Prove that for all  $n \in \mathbb{N}$ ,  $4 \nmid (n^2 + 2)$ .

See Ch 6, Ex 17. Can do by cases or contradiction.

11. Prove  $2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 2$  for  $n \in \mathbb{N}$ .

Can multiply left side by  $1 = (2 - 1)$  like an example in class, or use binomial theorem. Or prove by induction: see Ch 10, Ex 5.

12. Prove  $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$  for  $n \in \mathbb{N}$ .

This one you basically have to use induction—see Ch 10, Ex 3.

13. Prove that  $\{12n : n \in \mathbb{Z}\} \subseteq \{2n : n \in \mathbb{Z}\} \cap \{3n : n \in \mathbb{Z}\}$ .

This is Ch 8, Ex 1.

14. Prove that  $\{4k + 5 : k \in \mathbb{Z}\} = \{4k + 1 : k \in \mathbb{Z}\}$ .

This is Ch 8, Ex 26, and like examples we did in class. Suggestion: prove one side is a subset of the other and vice versa (“Method 1” for proving  $A = B$ ).