Here are some supplementary practice problems for the final exam. **Warning: this is not necessarily a comprehensive or representative set of review questions.** In particular, I tried to avoid too much repetition from Exams 1 and 2, the review problems for Exam 2, and the final homework (HW 7). You should definitely make sure you are comfortable with those problems as well.

1 True/False — no work needed

(this is a modified version of an old worksheet)

Notation: A, B, C denote sets; P, Q denote statements; $f : \mathbb{R} \to \mathbb{R}$ is a function.

1. T F $\exists A, \emptyset \in A$
2. T F $\forall A, \emptyset \in A$
3. T F $\exists A, \emptyset \subseteq A$
4. T F $\forall A, \emptyset \subseteq A$
5. T F $A \times B = B \times A$
6. T F if $A \times C = B \times C$, then $A = B$
7. T F $A \subseteq B \iff \mathcal{P}(A) \subseteq \mathcal{P}(B)$
8. T F $A \cup (B \cap C) = (A \cup B) \cap C$
9. T F $\mathbb{R} \subseteq \mathbb{R}^2$
10. T F $(\mathbb{Z} \times \mathbb{Z}) - (\mathbb{N} \times \mathbb{N}) = (\mathbb{Z} - \mathbb{N}) \times (\mathbb{Z} - \mathbb{N})$
11. T F $\sim (P \implies Q) \ = \ (\sim P \implies \sim Q)$
12. T F $P \implies Q \ = \ (\sim P) \vee Q$
13. T F $(P \implies Q) \ = \ (\sim Q \implies \sim P)$
14. T F $(P \wedge Q) \implies \sim (P \implies Q)$
15. T F $(P \iff Q) \implies (Q \implies P)$
16. T F $\sim (P \wedge Q) \ = \ (\sim P) \wedge (\sim Q)$
17. T F $\sim (\forall x > 0, f(x) > 0) = \forall x > 0, f(x) \leq 0$
18. T F $\sim (\exists x > 0, f(x) \in \mathbb{Q}) = \forall x > 0, f(x) \notin \mathbb{Q}$
19. T F $|(0, 1)| \neq |(0, \infty)|$.
20. T F $|\mathbb{N}| = |\mathbb{N} \times \mathbb{N}|$.

2 Problems

1. Give an example of a sentence which is not a statement.
2. Prove or disprove: every function from $A = \{1, 2\}$ to $B = \{1, 2, 3\}$ is injective.
3. Prove or disprove: no function from $A = \{1, 2\}$ to $B = \{1, 2, 3\}$ is surjective.
4. Show $x^2 = 1 - x^4$ has no solutions in $\mathbb{Z}$.
5. Show that if x is irrational, so is $\sqrt{x}$.
6. Prove that an integer is divisible by 2 if and only if its last digit is.
7. Let $A = B = \{1, 2, 3, 4\}$. How many functions are there $f : A \to B$?
8. Let $A = B = \{1, 2, 3, 4\}$. How many injective functions are there $f : A \to B$?
9. Let $A = B = \{1, 2, 3, 4\}$. How many surjective functions are there $f : A \to B$?
10. Let $A = B = \{1, 2, 3, 4\}$. How many bijective functions are there $f : A \to B$?
11. What is the coefficient of $x^{97}y^3$ in $(x + y)^{100}$?
12. Negate the statement: if $x^2 > 1$, then $x > 1$. Is this statement or its negative true?
13. Let A, B be sets in a universal set X. Prove or disprove: $\overline{A} \cup \overline{B} = \overline{A \cup B}$.
14. Show $a \in \mathbb{Z}$ is odd if and only if $a^2 + 2a + 3$ is even.
15. Prove $n^2 \leq n^3$ for all $n \in \mathbb{N}$.
16. Prove $3^n \geq 2^n + 1$ for all $n \in \mathbb{N}$.
17. Prove or disprove: if A and B are sets, then $\mathcal{P}(A) - \mathcal{P}(B) \subseteq \mathcal{P}(A - B)$.
18. Consider a 5-card hand dealt from a standard 52-card deck. How many hands are there such that:

    (a) there are at least 2 cards from 1 suit?

    (b) there are at least 2 cards which are clubs?

    (c) all cards are clubs?

    (d) all cards are clubs but non-consecutive? (a flush in clubs, but not a straight flush—recall if your cards are 2 3 ... 10 J Q K A, then you can think of J as 11, Q as 12, K as 13 and A can be either 1 or 14)
19. Prove or disprove: if A, B, C, D are sets, then $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$.
20. Prove $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)} = 1 - \frac{1}{n+1}$.
21. Give 2 infinite sets with the same cardinality, and 2 infinite sets with different cardinalities. For the 2 sets with the same cardinality, prove they have the same cardinality.
22. (Bonus) Explain Russell's paradox. What does it mean for set theory?