

Here are some review questions for you to try *before class* Friday, Nov 18, which you may solve by any method we've covered so far. We'll go over these in groups in class Friday, and by Saturday I will try to post some comments on these questions on the exams page.

Disclaimer: While there are more questions on here than will be on the exam, not all methods/concepts/topics you might need/want to use on the exam are necessarily covered here. I chose the questions here somewhat at random (though with more of an emphasis on induction proofs than may appear on the exam, as there was no homework on induction), and many if not all of the problems can be solved in multiple ways.

Questions:

1. Find and prove a formula for the alternating sum of the n -th row of Pascal's triangle.
2. Find and prove a formula for the sum of the first n rows of Pascal's triangle.
3. Prove $\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}$ for $n \in \mathbb{N}$.
4. Prove $F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}$, where F_n is the n -th Fibonacci number (initialized so $F_1 = F_2 = 1$).
5. Prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.
6. Prove that $\gcd(n, n+1) = 1$ for all $n \in \mathbb{N}$.
7. Prove the $6|(n^3 - n)$ for any $n \in \mathbb{N}$.
8. Prove that for all $n \in \mathbb{N}$, $4 \nmid (n^2 + 2)$.
9. Prove that $\{4k + 5 : k \in \mathbb{Z}\} = \{4k + 1 : k \in \mathbb{Z}\}$.
10. How many ways are there to order the letters ABCDE such that
 - (a) the two vowels are not adjacent? or
 - (b) all three consonants are not in a row?