Linear Algebra (MATH 3333) Spring 2009 Section 2
Midterm Practice Problems

Throughout the exam, V denotes a vector space.

1. Go over the problems on your quiz and previous homeworks, and make sure you can do them correctly.

True/False

Circle T or F.

2. T F Any two nonzero vectors in $\mathbb{R}^3$ are linearly independent.

3. T F A minimal spanning set for V is a basis for V.

4. T F Any subspace of $\mathbb{R}^2$ is either a line through the origin or $\mathbb{R}^2$.

5. T F The span of two nonzero vectors is either a line through the origin or a plane through the origin.

6. T F The set of polynomials in x of degree at most 5 form a vector space.

Definitions

7. If $S = \{v_1, v_2, \ldots, v_k\} \subseteq V$, define span($S$).

8. With S as above, define what it means for S to be a basis of V.

9. With S as above, define what it means for S to be linearly independent.

Problems

Show your work (i.e., prove your answers except where stated otherwise).

10. Show that the set of vectors of the form $\begin{pmatrix} a \\ b \\ a+b \end{pmatrix}$ in $\mathbb{R}^3$ forms a subspace of $\mathbb{R}^3$. Find a basis for this space (no proof needed). Describe this space geometrically.

11. Do the same as the previous problem for the subset $\{(x, y, z) : x + y + z = 0\}$ of $\mathbb{R}^3$.

12. Find two different bases for $\mathbb{R}^2$ (no proof needed).

13. Is $\{(x, y, z) : 2x - 3y + z = 1\}$ a subspace of $\mathbb{R}^3$?

14. Solve the system of equations or conclude that no solutions exist:

$$\begin{aligned} x - y &= 1 \\ 2x - y - z &= 1 \\ -x + 2y - z &= 1. \end{aligned}$$

15. Solve the system of equations or conclude that no solutions exist:

$$\begin{aligned} -2x + y + z &= 1 \\ x + z &= 0 \\ x + y - 2z &= -1. \end{aligned}$$

16. Is $\left\{\begin{pmatrix}1\\1\\-1\end{pmatrix},\begin{pmatrix}1\\-1\\1\end{pmatrix},\begin{pmatrix}-1\\1\\1\end{pmatrix}\right\}$ linearly independent? If not, find a maximal linear independent subset.

17. Do the same as the previous problem for $\left\{\begin{pmatrix}1\\2\\-2\\3\end{pmatrix},\begin{pmatrix}1\\-2\\-2\\3\end{pmatrix},\begin{pmatrix}2\\1\\-1\\0\end{pmatrix},\begin{pmatrix}-1\\1\\1\\-1\end{pmatrix}\right\}$.

17. Is $\left\{\begin{pmatrix}1\\2\\-2\end{pmatrix},\begin{pmatrix}1\\-2\\3\end{pmatrix},\begin{pmatrix}1\\-1\\0\end{pmatrix},\begin{pmatrix}-1\\1\\-1\end{pmatrix}\right\}$ a basis for $\mathbb{R}^3$?

18. Is $\left\{\begin{pmatrix}1\\1\\-1\end{pmatrix},\begin{pmatrix}1\\-1\\1\end{pmatrix},\begin{pmatrix}-1\\1\\1\end{pmatrix}\right\}$ a basis for $\mathbb{R}^3$?

19. Find a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ whose image is the set of vectors of the form $\begin{pmatrix}a\\2b-a\\b\end{pmatrix}$.