Linear Algebra (MATH 3333) Spring 2009 Section 2
Final Exam Practice Problems

Instructions: Try the following on your own, then use the book and notes where you need help. Afterwards, check your solutions with mine online. For Sections 1 and 2, no explanations are necessary. For all other problems, justify your work. There are also bonus problems at the end.

Note: Not every problem on the practice sheet is modeled off of one of your problems for homework. However, you can figure out how to do them with an understanding of the basic concepts from the course. They are designed to help piece together your understanding of the course material. There may be questions

1 True/False

In this section A is an $n \times n$ matrix.

1. T F Two vectors are linearly independent if they are not scalar multiples of each other.

2. T F Every square matrix is diagonalizable.

3. T F If A is diagonalizable, then there is a basis of eigenvectors of A.

4. T F If λ is an eigenvalue for A, then the eigenspace V_λ is a line.

5. T F If $A = PDP^{-1}$, then $A^3 = P^3D^3P^{-3}$.

6. T F $A = P_{T\leftarrow S}[A]_T P_{S\leftarrow T}$ where S is the standard basis for R^n.

7. T F Any linear transformation $A : \mathbb{R}^2 \to \mathbb{R}^2$ can be composed from (as a matrix, is a product of) rotations, reflections and scalings.

8. T F If A is 3×3 with real entries, it cannot have 3 distinct purely imaginary eigenvalues.

2 Short Answer

9. State the definition of a basis for a finite-dimensional vector space V.

10. State the definition of an eigenvalue and an eigenvector for an $n \times n$ matrix A.

11. What is the geometric meaning of the λ-eigenspace, V_λ, for A?

12. State three things linear algebra has applications to.

13. What is the geometric meaning of a linear transformation $A : \mathbb{R}^3 \to \mathbb{R}^3$?

14. What is the geometric significance of $\det(A)$ for a 3×3 matrix A?

15. In studying linear transformations, why does it suffice to study square matrices? Specifically, if $A : \mathbb{R}^2 \to \mathbb{R}^3$ and $B : \mathbb{R}^3 \to \mathbb{R}^2$ are two linear transformations, explain how to study them in terms of square matrices (or equivalently, linear maps from $\mathbb{R}^n \to \mathbb{R}^n$).

16. What is a Markov process? Give an example. What is the significance of the dominant eigenvectors?

17. Why might you want to exponentiate a matrix?

18. If A is a 2×2 matrix such that $A\begin{pmatrix}1\\2\end{pmatrix} = 4\begin{pmatrix}1\\2\end{pmatrix}$ and $A\begin{pmatrix}3\\-1\end{pmatrix} = 4\begin{pmatrix}3\\-1\end{pmatrix}$, what is A?

3 Problems

19. Let $A = \begin{pmatrix} 0 & 0 & -2 \\ 0 & -2 & 0 \\ -2 & 0 & 3 \end{pmatrix}$.

(i) Find the eigenvectors and eigenvalues of A.
(ii) Diagonalize A, i.e., write $A = PDP^{-1}$ for some diagonal matrix D.

20. (i) Construct a matrix A which rotates $\mathbb{R}^3$ by $\frac{\pi}{4}$ radians counterclockwise around the z-axis.
(ii) Construct a matrix B which rotates $\mathbb{R}^3$ by $\frac{\pi}{4}$ radians counterclockwise around the line $x = y = z$.

21. Suppose you have a (discrete) dynamical system given by

$$\begin{aligned} x(t+1) &= x(t) + 2y(t) \\ y(t+1) &= 4x(t) + 3y(t), \end{aligned}$$

with initial conditions $x(0) = 2$, $y(0) = 1$. Find explicit formulas for $x(t)$ and $y(t)$.

22. Go through Homework 7–12 and make sure you can do each problem, particularly HW 12 since it will not be graded before your final. (Problem A of HW 8 and Problem C of HW 9 are not so important. Most of the others are what I consider to be important material for the course, and you should expect several questions like these on the final.)