Linear Algebra (MATH 3333) Fall 2007 Sections 1/4
Homework 4

Due: Fri. Sept. 14, start of class

Instructions: You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top.

Conceptual Questions

1. Why do we want to determine the linear subspaces of $\mathbb{R}^n$?
2. What can the image of a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^3$ look like? From $\mathbb{R}^3 \to \mathbb{R}^2$?

Written Assignment

Section 4.1 (p. 188): 20, 21, 22

Problem A. Recall we defined a linear subspace of $\mathbb{R}^n$ to be the image of some linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$. From this definition, prove the following are subspaces of $\mathbb{R}^3$:
i) $\{(0, y, 0)\}$,
ii) $\{(0, 0, z)\}$,
iii) $\{(x, 0, z)\}$,
iiii) $\{(0, y, z)\}$.

Problem B. Determine the lines passing through p_1 and p_2 when
i) $p_1 = (3, 1)$ and $p_2 = (-1, 2)$ in $\mathbb{R}^2$,
ii) $p_1 = (1, 0, 3)$ and $p_2 = (2, -1, 4)$ in $\mathbb{R}^3$.

Problem C. Prove Theorem 1 in full generality, i.e., write

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix},$$

and prove that

$$A(cx + y) = cA(x) + A(y)$$

for a real number c.

Problem D. Determine all linear subspaces of $\mathbb{R}^3$ which are contained in the closed ball of radius 1 $\{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}$. Justify your answer.