Linear Algebra (MATH 3333) Fall 2007 Sections 1/4
Homework 3

Due: Fri. Sept. 7, start of class

Instructions: Please read the homework policies and guidelines posted on the course webpage. You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Page/section numbers refer to the course text.

Written Assignment

I encourage you to try think about the different proof methods in class (principally direct matrix computations and thinking in terms of linear transformations) for the problems below. One method may be better for some problems, but it may be worse or not work at all for others. Note A^2 denotes AA, A^3 denotes AAA, and so on.

Section 1.4 (pp. 40–41): 8, 10, 11, 34

Problem A. Prove algebraically that

$$(kA)B = k(AB) = A(kB)$$

for any $n \times n$ matrices A and B. You may use Propositions 1-3 and the Corollary from class (not all are necessary). As I did in class for the Corollary, at each step, state what result you are using. I will recall the results below for your convenience.

Prop. 1 $IA = A = AI$, where A and I are $n \times n$ matrices.

Prop. 2 $(kI)A = kA = A(kI)$, where A and I are $n \times n$ matrices.

Prop. 3 $(AB)C = A(BC)$ for any three matrices A, B, C of appropriate sizes such that the matrix multiplication is defined.

Cor. $(kA)B = A(kB)$, where A and B are $n \times n$ matrices.

Bonus

Bonus 1. Note that, as you might guess, the Corollary as well as the result of Problem A are true for A and B any $n \times m$ and $m \times \ell$ matrices, but we assume A and B are both $n \times n$ so we can use Propositions 1 and 2. Prove the Corollary and the result of Proposition A in the case where A is an $n \times m$ matrix and B is $m \times \ell$.