# Linear Algebra (MATH 3333) Fall 2007 Sections 1/4
# Homework 2

Due: Fri. Aug. 31, start of class

**Instructions:** Please read the homework policies and guidelines posted on the course webpage. You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top. Page/section numbers refer to the course text.

## Conceptual Questions (not to be turned in)

1. Why do we want to be able to write down a matrix for rotation by $\theta$ or reflection about $y = mx$?

2. What does the matrix $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ do?

## Written Assignment

**Problem A.** Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation. Prove that $T(0,0) = (0,0)$. (Write down the general form of a linear transformation $T(x,y)$ and plug in $x = 0$ and $y = 0$.)

**Problem B.** Write a matrix for the following transformations of $\mathbb{R}^2$:
(i) counter-clockwise rotation by 30 degrees
(ii) clockwise rotation by 45 degrees
(iii) reflection about $y = -2x$
(iv) reflection about $x = 0$

**Problem C.**
(i) Recall that reflection of $\mathbb{R}^2$ about the line $y = mx$ is given by $\frac{1}{m^2+1}\begin{pmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{pmatrix}$ (note this formula is also valid for $m = 0$, even though we assumed $m \neq 0$ when we derived this). Check that the determinant of reflection about $y = mx$ is -1 as asserted in class.
(ii) There is one other reflection about a line through the origin: the one about $x = 0$. Check this transformation also has determinant -1.

**Problem D.**
(i) Let $T_1(x,y) = (ex + fy, gx + hy)$ and $T_2(x,y) = (ax + by, cx + dy)$. Compute the composition $(T_2 \circ T_1)(x,y) = T_2(T_1(x,y))$ *without* using matrices.
(ii) Now writing $T_1$ and $T_2$ as matrices, compute the matrix multiplication $T_2T_1$ and show that this product is the composition $T_2 \circ T_1$ from part (i).

**From the text:**

**Section 1.3 (p. 31):** 11(a)(b), 12(b)(c), 14(a)(b), 18, 20