Linear Algebra (MATH 3333) Fall 2007 Sections 1/4
Homework 16 — Practice Problems

Instructions: Try the following on your own, then use the book and notes where you need help. You will be able to check solutions online and bring any questions you may have to the last class. For Sections 1 and 2, no explanations are necessary. For all other problems, justify your work. There are also bonus problems at the end.

1 True/False

1. T F Two vectors are linearly independent if they are not scalar multiples of each other.
2. T F Every square matrix is diagonalizable.
3. T F If A is diagonalizable, then there is a basis of eigenvectors of A.
4. T F If λ is an eigenvalue for A, then the eigenspace V_λ is a line.
5. T F If $A = PDP^{-1}$, then $A^3 = P^3D^3P^{-3}$.
6. T F $A = P_{T\leftarrow S}[A]_T P_{S\leftarrow T}$.

2 Short Answer

7. State the definition of a basis for a finite-dimensional vector space V.
8. State the definition of an eigenvalue and an eigenvector for an $n \times n$ matrix A.
9. What is the geometric meaning of the λ-eigenspace, V_λ, for A?
10. If A is a 2×2 matrix such that $A\begin{pmatrix}1\\2\end{pmatrix} = 4\begin{pmatrix}1\\2\end{pmatrix}$ and $A\begin{pmatrix}3\\-1\end{pmatrix} = 4\begin{pmatrix}3\\-1\end{pmatrix}$, what is A?
11. State three things linear algebra has applications to.

3 Problems

12. Let $A = \begin{pmatrix}0 & 0 & -2\\0 & -2 & 0\\-2 & 0 & 3\end{pmatrix}$.
    (i) Find the eigenvectors and eigenvalues of A.
    (ii) Diagonalize A, i.e., write $A = PDP^{-1}$ for some diagonal matrix D.
13. Suppose you have a (discrete) dynamical system given by

$$\begin{aligned} x(t+1) &= x(t) + 2y(t) \\ y(t+1) &= 4x(t) + 3y(t), \end{aligned}$$

with initial conditions $x(0) = 2$, $y(0) = 1$. Find explicit formulas for $x(t)$ and $y(t)$.

14. Go over Homeworks 11–15 and Exam 2, and make sure you can do the problems correctly on your own. There will be similar problems on the final.
15. If you are interested in two drums which sound alike but have different shapes, go here:
    http://www.ams.org/featurecolumn/archive/199706.html
    There is a link to an animation at the bottom.

Bonus

You may do any of the following for bonus credit. You may turn in your solution in to me any time before the end of the semester.

Bonus 1. (Google Pagerank) Do Example 11 on p. 513. You should use a computer program (e.g., Mathematica or Maple) which will calculate eigenvalues and eigenvectors for you. Find all the eigenvalues, identify the dominant one, then find a dominant eigenvector, and use this to rank the websites. You should turn in an explanation of your work together with a printout of your calculations.

Bonus 2. Let A be a 2×2 matrix. Prove there is a basis $T = \{v_1, v_2\}$ of $\mathbb{R}^2$ such that $[A]_T$ is either diagonal or of the form $\begin{pmatrix} \lambda & x \\ 0 & \lambda \end{pmatrix}$. In the first case, we know how to use $[A]_T$ to compute $A^1 00$. In the second case, explain how one can do the same thing. (Compute $\begin{pmatrix} \lambda & x \\ 0 & \lambda \end{pmatrix}^{100}$.)