# Linear Algebra (MATH 3333) Fall 2007 Sections 1/4
# Homework 15

Due: Mon. Dec. 3, start of class

**Instructions:** You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top.

## Reading

Sections 7.2

## Conceptual Questions

1. How can we compute high powers of matrices? Why do we want to?

## Written Assignment

18 points

**Problem A.** (6 pts) Let

$$A = \frac{1}{2}\begin{pmatrix} 1 & 3 & -3 \\ 3 & 1 & -3 \\ 0 & 0 & 2 \end{pmatrix}.$$

(i) Find the eigenvalues and eigenvectors of $A$

(ii) Find a basis $T$ of $\mathbb{R}^3$ such that $[A]_T$ is diagonal.

(iii) Writing $A = P_{S\leftarrow T}[A]_T P_{T\leftarrow S}$ where $S$ is the standard basis, compute $A^{10}$.

**Problem B.** (4 pts) Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

(i) Prove $A$ is *not* diagonalizable.

(ii) While $A$ is not diagonalizable, it is close to being diagonal, and is called *(upper) triangular*. This matrix is still easy to exponentiate. By computing the general product

$$\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix},$$

determine $A^{137}$.

**Problem C.** (8 pts) Let $c(t)$ and $r(t)$ denote the number of coyotes and roadrunners in your backyard in $t$ years from now. Suppose they satisfy the (oversimplified and bad) predator-prey relations $c(t+1) = 0.5c(t) + 0.25r(t)$ and $r(t+1) = -0.5c(t) + 1.25r(t)$.

(i) Find formulas for $c(t)$ and $r(t)$ in terms of the initial populations $c(0)$ and $r(0)$.

(ii) In particular, what will the population of coyotes and roadrunners be in 5 years if $c(0) = 10$ and $r(0) = 100$. What is the long term tendency of this system, i.e., what is the limit of $c(t)$ and $r(t)$ at $t \to \infty$?