Linear Algebra (MATH 3333) Fall 2007 Sections 1/4
Homework 13

Due: Mon. Nov. 19, start of class

Instructions: You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top.

Reading

Sections 6.5, 7.1

Conceptual Questions

1. Why are we looking for eigenvectors and eigenvalues?
2. What is the geometric meaning of v being an eigenvector for A with eigenvalue λ?

Written Assignment

22 points

Section 6.5 (p. 413): 3 (4 pts)

Problem A. (6 pts) Let $A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$. Consider the bases $S = \{e_1, e_2\}$ and $T = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$. Find

(a) $P_{S \leftarrow T}$
(b) $P_{T \leftarrow S}$
(c) $[A]_T$, the matrix for the linear transformation A with respect to T, using the transition matrices from (a) and (b).

Problem B. (12 pts) Find the eigenvalues for each of the six (6) matrices in #7 and #9(a)(b) of Section 7.1 (p. 450).

Bonus. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Suppose v is an eigenvector for A with eigenvalue λ.

(a) Show this is equivalent to $(\lambda I - A)v = 0$.
(b) Show this is equivalent to the matrix $\lambda I - A$ being not invertible.
(c) Using the result of last week's bonus question (whether or not you did it), conclude that the eigenvalues are indeed the roots of the characteristic polynomial $\det(\lambda I - A)$ as asserted in Theorem 7.1.