Linear Algebra (MATH 3333) Fall 2007 Sections 1/4
Homework 12

Due: Fri. Nov. 9, start of class

Instructions: You may **not** use a calculator (or computer). Make sure to write your name, course and section numbers in the top right corner of your solution set, as well as the assignment number on top.

Reading

Sections 6.3, 4.8, 2.3

Written Assignment

34 points

Section 6.3 (p. 397): 1(a)(b)(d) (6pts)

Section 2.3 (pp. 124–125): 9 (8pts) *Note:* singular *means not invertible.*

Problem A. (8 pts) Let $A = \begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 3 & 1 & 1 & 1 \end{pmatrix}$. Find

(a) $\operatorname{rank}(A)$;
(b) $\operatorname{nullity}(A)$;
(c) a basis for $\operatorname{image}(A)$;
(d) a basis for $\ker(A)$.

Problem B. (12 pts) Let $S = \{e_1, e_2, e_3\}$ and $T = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\}$ be bases for $\mathbb{R}^3$.

(a) Find $[e_1]_T$, $[e_2]_T$ and $[e_3]_T$.

(b) If $[v]_T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, find v.

(c) Find $P_{S \leftarrow T}$.

(d) Repeat (b) using $P_{S \leftarrow T}$.

(e) The transition matrix $P_{T \leftarrow S}$ is the 3×3 matrix whose i-th column is $[e_i]_T$ (cf. p. 261). Check that the matrix products $P_{T \leftarrow S} P_{S \leftarrow T}$ and $P_{S \leftarrow T} P_{T \leftarrow S}$ both equal the identity matrix I (i.e., $P_{T \leftarrow S} = P_{S \leftarrow T}^{-1}$).

(f) Use $P_{T \leftarrow S}$ to find $[v]_T$ where $v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

Bonus. #21, p. 125.