Linear Algebra (MATH 3333) Fall 2007 Sections 1/4
Homework 10 — Practice Problems

Not to be turned in

Instructions: Try the following on your own, then use the book and notes where you need help. You will be able to check solutions online and bring any questions you may have to Wednesday's class. For Sections 1 and 2, no explanations are necessary. For all other problems, justify your work.

In the questions below, V denotes a finite-dimensional vector space.

1 True/False

1. T F Any set of linearly independent vectors for V contains a basis for V.
2. T F Any subspace of $\mathbb{R}^3$ of dimension 2 is a plane through the origin.
3. T F Two vectors are linearly independent if they are scalar multiples of each other.
4. T F Any set of vectors containing the zero vector is linearly dependent.
5. T F P_3 is isomorphic to $\mathbb{R}^3$.

2 Short Answer

6. State the definition of a vector space.
7. State the definition of a basis for V.
8. State the definition of linear independence of a set $v_1, \ldots, v_k$ of vectors in V.
9. State the definition of the dimension of V.
10. Determine the rank of the following matrices:

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}, \quad \begin{pmatrix} 2 & 3 & -1 \\ 1 & 2 & -2 \\ -1 & -3 & 5 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & -1 \\ 1 & 9 & -1 \\ -3 & 8 & 3 \\ -2 & 3 & 2 \end{pmatrix}$$

3 Problems

11. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$. Determine the image of A in terms of span. What is its dimension?

12*. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$. Determine the kernel of A. What is its dimension?

13*. Let A be a linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$. If the kernel of A is a line, what is the rank of A?

14. Go over Homeworks 6–9, and make sure you can do the problems correctly on your own. There will be similar problems on the exam.

* #12 and #13 require material from Monday's lecture.

Bonus. Prove any subspace of $\mathbb{R}^n$ (a vector space) is a linear subspace of $\mathbb{R}^n$ (the image of a linear transformation).